Supplementary Material

"High Fequency GaAs nano-optomechanical disk resonator"

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1) Effect of the fluctuation of gap distance between disk and fibre.

The results presented in the article rely on the assumption that a mechanical displacement $\Delta\alpha$ of the disk causes a change in the Whispering Gallery Mode (WGM) optical resonance via the change of the disk resonator geometry. However, a change in the transmission ΔT could also occur because the distance between the disk and the tapered fibre varies upon mechanical displacement of the disk, producing a change in the disk-fibre optical coupling. We provide here supplementary data and calculations showing that we can safely ignore this effect in the present study. The optomechanical effects presented in the article are solely to be assigned to a change of the cavity geometry.

a) First complementary experiments:

We verify here that the measured ΔT is indeed induced by a mechanical displacement $\Delta\alpha$ of the resonator with a relation of $\Delta T=(dT/d\omega)(d\omega/d\alpha)\Delta\alpha=(dT/d\omega)g_{OM}\Delta\alpha$. In the (tapered fibre-disk) optical transmission spectrum at low power level (for example in the inset of Fig1 of this Supplementary Material, the grey curve at 6 μW of optical power measured on the photodetector), a WGM resonance has a maximum of $dT/d\omega$ at the inflection point of the flank of the resonance, while $dT/d\omega$ is strictly zero on resonance. This indicates a maximum sensitivity of our optomechanical measurement of the mechanical motion at the inflection point.

To verify this assertion, we measure the RF noise spectrum of the transmission as a function of the optical wavelength in the vicinity of the resonance of a given WGM. Figure 1 of Supplementary Material shows the result of such measurement: the optically measured mechanical spectrum around one given mechanical resonance (mode A of the article) as a function of optical wavelength (see color points in the inset). In the spectrum, the amplitude of the mechanical resonance first increases when increasing wavelength from the off-resonance condition (blue detuned to resonance) towards on-resonance condition. This amplitude reaches a maximum at 1542.51 nm around the inflection point and then decreases when continuing scanning up the wavelength towards the resonance peak. The amplitude reduces very quickly after 1542.58 nm. This corresponds to the expected behavior of a maximal sensitivity at the inflection point. The change of mechanical resonance peak amplitude with respect to optical power is negligible in the scale of the figure (the figure is in log scale).

In case of a measurement dominated by the modulation of the gap distance between disk and fibre, the amplitude of the mechanical peak in the spectrum would simply scale with the amount of photons exiting the fibre output port. This is clearly not what is observed in Fig1.

b) Second complementary experiments:

We bring the fibre-taper directly in physical contact to the disk. The fibre remains stick to the disk by an electrostatic force: the gap distance is constant in that case and cannot fluctuate upon disk motion. Under these conditions, we measure the disk mechanical resonances in the transmission RF spectrum: the resonances have the same amplitude as under normal conditions, and the value of the optomechanical coupling g_{OM} extracted from the measurement remains also the same. This conversely shows that the effect of gap distance fluctuation is negligible under normal conditions, when fibre and disk are separated.

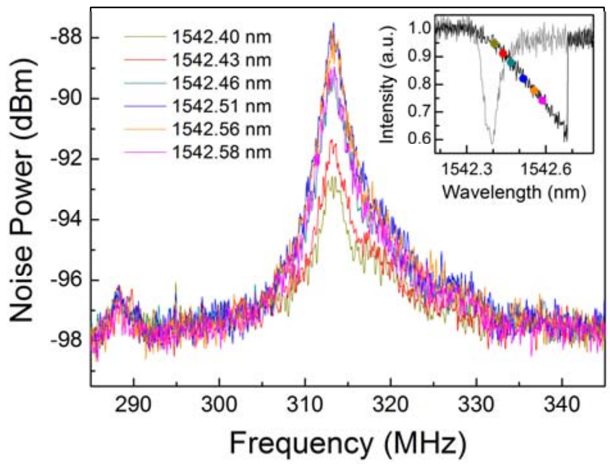

Figure 1*: A mechanical resonance of a GaAs disk with a radius of 5 mm is measured as a function of optical wavelength in the vicinity of a given WGM resonance (inset, black curve). Inset gives the optical transmission spectrum of the WGM at low (grey) and high (black) optical power. Different wavelengths are indicated with corresponding colors.*

c) Complementary calculation:

We investigate the effect of gap-distance modulation using the theoretical description of the disk-fibre evanescent coupling problem, based on coupled modes theory. Using the main article notation and noting g the gap distance, we present here the analytical calculation of $dT/d\alpha=(dT/d\omega)g_{OM}$ and dT/dg, and compare the order of magnitude of both terms. Considering the coupling between a straight single mode waveguide and a disk resonator WGM, the transmission of the waveguide is given by [1]

$$T(\Delta\omega, g) = \left|\frac{[\gamma_i - \gamma_e(g)]/2 + i\Delta\omega}{[\gamma_i + \gamma_e(g)]/2 + i\Delta\omega}\right|^2 = \frac{\{[\gamma_i - \gamma_e(g)]/2\}^2 + \Delta\omega^2}{\{[\gamma_i + \gamma_e(g)]/2\}^2 + \Delta\omega^2}.$$

where $\Delta\omega$ is the laser frequency detuning to the WGM resonance frequency ω_0, g is the disk-waveguide gap distance, $\gamma_e(g)$ is the extrinsic coupling rate (the WGM loss rate induced by the coupling to the fibre) and $\gamma_i=\omega_0/Q_i$ the intrinsic WGM cavity loss rate. The FWHM of the WGM resonance is $\delta\omega=\gamma_e(g)+\gamma_i$ and the loaded Q factor $Q_{load}=\omega_0/\delta\omega$. The transmission contrast is $\Delta T=T(\infty,g)-T(0,g)=4\gamma_i\gamma_e(g)/[\gamma_i+\gamma_e(g)]^2$.

(i) For arbitrary gap distance g and at the inflection point $\Delta\omega=[\gamma_i+\gamma_e(g)]/2\sqrt{3}$,

$$\left|\frac{dT}{d(\Delta\omega)}\right| = \frac{2|\Delta\omega|\gamma_i\gamma_e}{\left\{\left[(\gamma_i+\gamma_e)/2\right]^2+\Delta\omega^2\right\}} = \frac{3\sqrt{3}}{4}\Delta T\,\frac{Q_{load}}{\omega_0}\,.$$

Taking typical values for the GaAs disks investigated in the article, $\Delta T\sim1$, $Q_{load}\sim5*10^4$, $\omega_0=1.2*10^{15}$Hz, and $g_{OM}\sim100$GHz/nm, we obtain $dT/d\alpha\approx5\text{nm}^{-1}$.

(ii) The evanescent coupling parameter varies approximately exponentially with the gap distance g. Around a given gap distance g_0, it can be expanded as $\gamma_e(g)=\gamma_e(g_0)\exp(-\eta\Delta g)\approx\gamma_e(g_0)(1-\eta\Delta g)$ for small Δg (η is the decay constant). In this case:

$$\left|\frac{dT}{d(\Delta g)}\right| = \frac{4\eta}{\left[1+(2f)^2\right]}\frac{\gamma_e(g_0)}{\gamma_i}\frac{\left|1-\gamma_e(g_0)/\gamma_i\right|}{\left[1+\gamma_e(g_0)/\gamma_i\right]^3}\,,$$

where $f=\Delta\omega/\delta\omega=\Delta\omega/[\gamma_e(g_0)+\gamma_i]$, and $f\geq0$. When f=0 (on resonance), dT/dg is maximum. If g_0 chosen to have critical coupling $\gamma_e=\gamma_i$, then dT/dg=0, that is the effect of transmission modulation by the fluctuation of g is strictly zero. Even in a situation where critical coupling is not obtained (a typical case would be a gap distance g_0 such that $T(0,g_0)=1/2$, corresponding to $\gamma_e(g_0)/\gamma_i=0.17$), dT/dg would still only amount to $\approx10^{-3}\text{nm}^{-1}$, considering a typical value of $\eta\sim1/400\ \text{nm}^{-1}$ for our study [2]. Even in this case, the transmission modulation induced by gap distance fluctuation is 3 orders of magnitude smaller than that due to the optomechanical coupling in the disk itself.

2) <u>On the Brownian nature of the measured disk mechanical motion</u>.

We show here that the disk mechanical motions measured in the main article are independent from the employed optical power: the measured vibration amplitudes are in the Brownian motion limit at room temperature here and not driven by an optical excitation.

Figure 2 of this Supplementary Material shows calibrated spectra of a disk mechanical motion, obtained at different optical powers using the optomechanical measurement presented in the main article. For clarity, the contribution from the laser shot-noise is removed here to let the mechanical displacement alone. These measurements are taken on one of the disks listed of the article (a disk with radius 4.6 µm). As can be seen on the figure, the amplitude of the mechanical motion is independent on the used optical power, and fits very well to the curve expected from equipartition theorem for the corresponding harmonic oscillator at 300K. This shows that in this the regime, the disk mechanical motion is merely thermally excited (Brownian motion) and not driven by optical excitation.

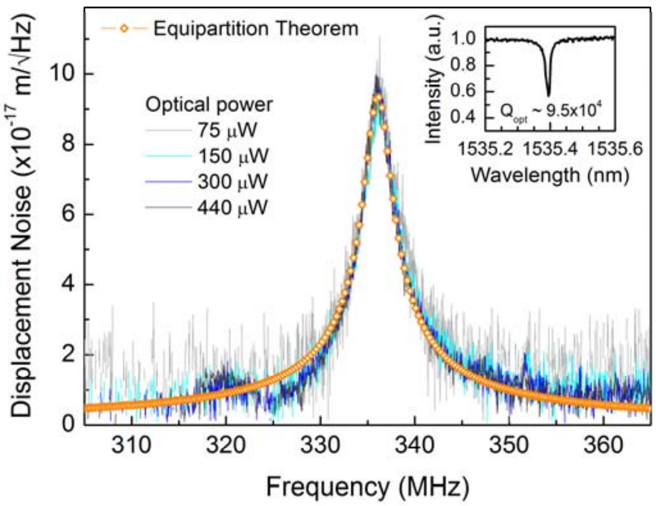

Figure 2: *Calibrated Brownian motion of a disk mechanical motion, probed at different optical powers. The indicated power is the value measured on the photodetector. Inset is the linear transmission spectrum of the WGM resonance chosen for the measurement.*

References:

[1] H. A. Haus "*Waves and Fields in Optoelectronics,*" (Englewood Cliffs, NJ: Prentice-Hall) (1984).

[2] L. Ding, P. Senellart, A. Lemaitre, S. Ducci, G. Leo, and I. Favero, "*GaAs micro-nanodisks probed by a looped fiber taper for optomechanics application,*" **Proc. SPIE**, Vol. 7712, 771211 (2010).