Injection locking of an electro-optomechanical device

CHRISTIAAN BEKKER, RACHPON KALRA, CHRISTOPHER BAKER,* AND WARWICK P. BOWEN

Centre for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, Australia
*Corresponding author: c.baker3@uq.edu.au

Received 10 July 2017; revised 31 August 2017; accepted 31 August 2017 (Doc. ID 301920); published 29 September 2017

Advances in optomechanics have enabled significant achievements in precision sensing and control of matter, including detection of gravitational waves and cooling of mechanical systems to their quantum ground states. Recently, the inherent nonlinearity in the optomechanical interaction has been harnessed to explore synchronization effects, including the spontaneous locking of an oscillator to a reference injection signal delivered via the optical field. Here, we present, to the best of our knowledge, the first demonstration of a radiation-pressure-driven optomechanical system locking to an inertial drive, with actuation provided by an integrated electrical interface. We use the injection signal to suppress the drift in the optomechanical oscillation frequency, strongly reducing phase noise by over 55 dBc/Hz at 2 Hz offset. We further employ the injection tone to tune the oscillation frequency by more than 2 million times its narrowed linewidth. In addition, we uncover previously unreported synchronization dynamics, enabled by the independence of the inertial drive from the optical drive field. Finally, we show that our approach may enable control of the optomechanical gain competition between different mechanical modes of a single resonator. The electrical interface allows enhanced scalability for future applications involving arrays of injection-locked precision sensors.

1. INTRODUCTION

The ability to engineer strong interactions between high-quality electromagnetic cavities and mechanical resonators has led to a rich variety of results in the field of optomechanics, including ground-state cooling [1,2], quantum-limited measurement [3,4], and optomechanical entanglement [5]. Optomechanical systems have further proven to be a powerful tool for applications involving precision sensing [6–8]. Minute perturbations in the physical environment can be detected through changes in the resonance frequency of a nanomechanical resonator, making these systems widely used as mass [9,10] and gas [11] sensors. Furthermore, many optomechanical systems can be fabricated on-chip, allowing dense arrays of sensors to be employed for applications such as multiparticle sensing for detection of lung cancer [12]. One important feature of optomechanical systems is the ability of the optical field to amplify mechanical motion. This regenerative amplification has been shown to allow enhanced sensitivity [13] and presents an avenue for the exploration of synchronization phenomena in optomechanical systems.

Nonlinear effects can cause two resonators to become synchronized, where the phases of their individual oscillations lock with respect to each other. A network of coupled oscillators can spontaneously synchronize, as was first reported for pendulum clocks hung from a common frame [14] and further observed in numerous biological systems, including the flashing of fireflies [15]. Alternatively, a single oscillator can synchronize to the phase of an externally applied drive through an effect known as injection locking, as has been studied extensively in the context of electrical tank circuits [16–18], implemented in nonlinear mechanical resonators [19], and further observed for the effect of light on human circadian rhythms [20]. Both these forms of synchronization have been explored in optomechanical systems. Arrays of optomechanical systems can synchronize when coupled through an overlap of their optical modes [21,22], via coupling to a common optical waveguide [23], or through direct inertial coupling [24,25]. Injection locking of a single optomechanical oscillator has also been demonstrated [26–29].

Conventionally, the injection signal is delivered to the optomechanical oscillator through the optical field, achieved experimentally via modulation of the input laser power [26–28]. Alternatively, as pointed out in a recent theoretical study [30], the oscillator can be driven with a direct inertial force. By evading the cavity filtering inherent to optical driving, this introduces different synchronization dynamics. Moreover, inertial drives can be locally applied to individual oscillators with integrated electrodes, thus presenting a scalable approach for applications utilizing arrays of oscillators. While direct inertial forcing has been achieved in a bolometric system [29], it has not been demonstrated previously with a radiation-pressure optomechanical device.
oscillator. The conservative nature of the radiation-pressure interaction, in contrast to bolometric systems, enables many of the most important applications of optomechanical systems in quantum science and technology [8].

In this work, we present the first demonstration of the locking of a radiation-pressure optomechanical oscillator to an inertial drive. We demonstrate the ability of the injection signal to suppress phase noise by over 55 dBc/Hz and to tune the oscillation frequency by more than 2 million times the oscillation linewidth, and explore previously unreported locking dynamics. With our approach, the inertial injection signal is the electrostatic force between two electrodes directly patterned onto the body of the resonator. The feed-forward stabilization achieved with this electrical injection locking may prove more scalable for sensor arrays or complex many-body quantum networks as compared with optical injection locking, or implementing feedback circuitry for individual free-running resonators [31]. For single-oscillator sensors, a potential application of the inertial drive is to employ anomalous cooling [32] to suppress undesired regenerative oscillation of certain mechanical modes, an issue encountered in precision optical sensors such as gravitational wave detectors [33–35].

Regenerative oscillation, often referred to as self-sustained oscillation, is maintained via an intrinsic feedback loop introduced by the radiation-pressure interaction, which induces dynamical back-action between the light in the optical cavity and the motion of the mechanical element (see Fig. 1). The synchronization phenomenon studied in this work is enabled by the inherent nonlinearity in the optomechanical interaction. As discussed above, all previous injection-locked radiation-pressure optomechanical oscillators have used optical injection, which is described by the same feedback loop [Fig. 1(a)] as studied widely in the context of injection-locked tank circuits [17]. In contrast, inertial injection bypasses the optical system response to drive the mechanical oscillator directly, qualitatively modifying the dynamical behavior of the system. This provides an opportunity to explore different synchronization behavior. Furthermore, in contrast to optical injection, the inertial force can be made larger than the radiation-pressure force. Our measurements uncover two different regimes of injection locking which display qualitatively different behavior when the resonator is partially locked. This may stem from the unique way in which our injection signal is applied.

2. ELECTRO-OPTOMECHANICAL SYSTEM

The electro-optomechanical system used in this work consists of a microtoroidal optomechanical oscillator [36,37] with an integrated electrical interface that allows a radial force to be applied directly to the mechanical resonator. In a previous work, this force was used to demonstrate high-bandwidth tuning of the optical resonance frequency [38]. The optical mode is a whispering gallery mode (WGM) of the silica microtoroid of radius 100 μm, described by its ladder operators $a$ and $a^\dagger$, with a resonance frequency of $\omega_m/2\pi \approx 194$ THz and a linewidth of $\kappa/2\pi \approx 100$ MHz. Figure 2(a) shows a false-color electron micrograph of the microtoroid device, consisting of a reflown silica disk (blue) atop an etched silicon pedestal (gray). Apart from having a larger radius, this device is identical to that studied in Ref. [38], which can be referred to for details on device fabrication. Measurements are performed by bringing the tapered section of an optical fiber in contact with the microtoroid to couple to the WGM while reducing taper drift. Laser light of frequency $\omega_L$ (wavelength of ~1550 nm) and power $P_{in}$ is delivered to one end of the fiber and the transmitted power through the taper, $P_{out}$, is measured by a photodetector, allowing the optical WGM to be probed. The optical measurement setup is shown in green in Fig. 2(a). All measurements are made in ambient conditions.

The optical mode couples to a mechanical radial breathing mode of the microtoroid, described by its ladder operators $b$ and $b^\dagger$. Figure 2(b) shows the result of a finite-element method (COMSOL) simulation of the mechanical mode with a frequency of $\omega_m/2\pi \approx 8.9$ MHz, effective mass of $m_{eff} \approx 30$ ng, and zero-point motion of $x_{zp} \approx 0.3$ fm. With $\omega_m/\kappa \approx 0.09$, the optomechanical system operates in the unresolved sideband regime. The strength of the optomechanical coupling is parameterized by the single-photon coupling rate, $g_0 = G x_{zp}$. $G/2\pi \approx 0.9$ GHz/nm, estimated by COMSOL simulations, is the amount by which the optical mechanical motion shifts the optical resonance frequency [6]. Detuning the laser frequency appropriately allows probing of the mechanical motion through its modulation of the transmitted optical power. The Hamiltonian in the frame rotating with the frequency of the laser drive, $\omega_L$, is
Here, $\Delta = \omega_L - \omega_1$ is the detuning and $A_L = \sqrt{\eta P_{in}/\hbar \omega_L}$ is the optical drive amplitude, where $\kappa_{in}$ is the optical coupling rate between the taper and the WGM and $\eta \approx 0.15$ captures the power lost from the laser to the taper. In addition to allowing the optical drive amplitude, where $\Delta$ governs its motion and an intrinsic mechanical linewidth (full width at half-maximum, or FWHM) of $\Gamma/2\pi \approx 15$ kHz can be measured. As $P_{in}$ increases, the mechanical mode narrows and is frequency-upshifted, consistent with optomechanical theory [6,8]. Once $P_{in}$ surpasses the threshold for regenerative oscillation, the oscillation amplitude increases by several orders of magnitude and the linewidth narrows beyond the frequency resolution of the spectrum analyzer, as shown for $P_{in} = 10$ mW. At this point, the amplitude is limited by the inherent nonlinearity in the optomechanical interaction which drives the oscillations and enables the synchronization effects studied in this work. This nonlinearity can be intuitively understood by considering the Lorentzian lineshape of the optical mode as a function of the detuning caused by mechanical displacement [6,8]. For small oscillation amplitudes, the power coupled into the cavity varies linearly with mechanical displacement. However, at larger amplitudes comparable to or greater than $\kappa/G$ that probe the Lorentzian shape of the mode, this is no longer the case. Any mechanical nonlinearity can be neglected given the relatively small oscillation amplitudes studied here, estimated to be $<100$ pm, several orders of magnitude smaller than the smallest feature in the device. Kerr and Raman optical nonlinearities can also be neglected as the incident powers used are orders of magnitude below the threshold required to observe these effects [40].

The electrical interface of the device allows an inertial force—which is completely independent of the optics—to be applied to the microtoroid [31,38]. This is achieved through the integration of a circular capacitor [highlighted in yellow in Fig. 2(a)] of capacitance $C \approx 7$ fF, whose attractive force occurs in the radial direction and, hence, has a strong overlap with the radial breathing mode. As shown in the figure, a circular slot is etched through the disk, between the gold electrodes, to increase mechanical compliance. A voltage bias is applied across the capacitor via tungsten probe tips. Baker et al. [38] studied the ability of an applied voltage to tune the optical resonance frequency and did not operate in a regime where optical amplification of the mechanical motion was appreciable. By contrast, here we seek to enhance the optomechanical gain by increasing the laser power inside the cavity and operating with a blue-detuned laser. This allows us to reach the regime of regenerative oscillation and, by applying an RF voltage with frequency $\omega_d \approx \omega_m$ study the synchronization of the mechanical oscillations.

We apply a drive voltage $V_d(t) = V_{DC} + V_{AC} \cos(\omega_d t)$. The DC and $2\omega_d$ components of the resulting attractive force between the capacitor plates (proportional to $V_{AC}^2$ [38]) are off-resonant and can be omitted to obtain the drive force $F_d(t) = (\delta C/\delta x) V_{DC} V_{AC} \cos(\omega_d t)$. The change in capacitance per unit mechanical displacement in our device is estimated to be $\delta C/\delta x \approx 4 \times 10^{-4}$ fF/nm from finite-element method simulations. The complete system Hamiltonian is then $H = H_{opt} + H_d$, where

$$H_d = x_{m} F_d(t)(b + b^\dagger).$$

As discussed in the previous section, we highlight that this form of a drive is distinct from previous demonstrations of locking in optomechanical systems, which used optical radiation-pressure modulation driving [26,28]. Although the two implementations require similar instrumentation overhead for the synchronization of a single oscillator, our electrical approach should be more scalable for applications involving arrays of oscillators. A single electrical drive can be straightforwardly distributed to all resonators on a chip, whereas the optical driving scheme would generally require one optical modulator per resonator due to mismatches.

---

**Fig. 3.** (a) Power spectra of the mechanical mode measured via its modulation of the transmitted optical power $P_{out}$ for $P_{in} = 4$ mW, 6 mW, 8 mW, and 10 mW. For each setting of $P_{in}$, the laser detuning (in the range of $\kappa/2$) is modified to maximize the mechanical modulation of $P_{out}$. At lower input powers $P_{in}$, the mechanical motion is dominated by thermal excitation. At 10 mW, regenerative oscillation is observed, marked by a significant increase in oscillation amplitude and linewidth narrowing. (b) Phase-noise measurement of the mechanical oscillations with $P_{in} = 10$ mW. A fit to the data shows that the linewidth of the regenerative oscillations is 30 mHz. Note that we add to our fit a Lorentzian peak at an offset of 220 Hz and the noise floor at $-120$ dBc/Hz.
in optical resonance frequencies. The electrical approach also benefits from \textit{in situ} tuning of the optical resonance frequency with a DC voltage [38].

3. INJECTION LOCKING

A. Locking and Stability

Figure 3(a) shows the power spectrum of the regeneratively oscillating mechanical mode. We see that, although the natural linewidth of the mechanical mode is 15 kHz, the optomechanical gain significantly narrows the linewidth such that it is no longer resolved by the spectrum analyzer. We thus use a phase-noise analyzer to capture the narrowed linewidth. Figure 3(b) shows the measured phase noise for the optomechanical oscillator with $P_{\text{in}} = 10 \text{ mW}$. The expected phase noise (units of dBc/Hz) for the oscillator is [41]

$$L(f) = 10 \log_{10} \left( \frac{1}{\pi (f_{\text{bw}})^2 + (\Delta f)^2} \right).$$

where $\Delta f$ is the frequency offset from the carrier and $f_{\text{bw}}$ is the half-width at half-maximum. The data in Fig. 3(b) are fit to the above equation, confirming the expected $1/(\Delta f)^2$ lineshape and yielding a linewidth of 30 mHz. This value is $5 \times 10^3$ times smaller than the intrinsic linewidth of the mechanical mode.

While the linewidth of the regenerative oscillations is extremely small, this particular measurement was performed with frequency offsets above 2 Hz and therefore does not capture the fact that the oscillation frequency drifts significantly in the timescale of minutes. This drift would be detrimental to the use of the oscillator for applications that require long-term frequency stability such as mass or temperature sensors. To highlight the broadening caused by this drift, we compare a single acquired power spectrum (light green) with the average of 50 traces acquired consecutively over a timescale of minutes (dark green) for $P_{\text{in}} = 20 \text{ mW}$ in Fig. 4(a). The oscillation frequency drifts over a range of $\approx 100$ Hz in this time, corresponding to more than 3000 times the reduced linewidth. This significant drift can be eliminated by applying the external drive described by Eq. (2) to which the mechanical oscillations can synchronize.

To demonstrate this effect, we apply a drive of $V_{\text{AC}} = 0.5 \text{ V}$ with $V_{\text{DC}} = 50 \text{ V}$ at a frequency of $\omega_{\text{dr}}/2\pi \approx 8.914$ MHz. Figure 4(a) shows a single trace (light blue) and the average of 50 acquired traces (dark blue) of the power spectrum of the locked oscillations. We observe that the peak of the averaged trace overlaps exactly with that of the single acquisition, indicating that the oscillations are indeed locked and that the frequency drift is eliminated. In addition to the elimination of frequency drift, a comparison of the single acquisitions of the locked (light blue) and unlocked (light green) traces reveals that the locking significantly reduces the quasi-instantaneous linewidth. This effect of injection locking is well known in other types of synchronized oscillators [18], and has been demonstrated before in optomechanics with a radiation-pressure-modulated drive [26]. It is also used, for instance, in laser systems to reduce the phase noise of a noisy high-power laser by locking it to a phase-stable low-power laser [42].

To explore this further, we measure the phase noise for the locked oscillations for varying drive strengths. Figure 4(b) shows plots of phase noise for $V_{\text{AC}} = 0.1 \text{ V}$, 0.5 V, and 1.5 V with $P_{\text{in}} = 10 \text{ mW}$. Also included in the figure is the phase-noise trace of the unlocked oscillations for comparison [reproduced from Fig. 3(b)], as well as the phase noise of the RF voltage source itself. We clearly observe a significant suppression of phase noise when the oscillator is locked, with a maximum suppression of over 55 dBc/Hz at 2 Hz. A stronger drive results in a greater phase-noise suppression, approaching the noise of the RF source itself [26]. As discussed in Ref. [18] in the context of electrical tank circuits, and as shown in Fig. 4(b), the phase noise is suppressed up to a critical frequency offset which depends on the drive strength. Above that frequency, the phase noise of the unlocked or free-running oscillator is recovered. As we will see in the next section, where injection locking is used to tune the frequency of regenerative oscillations, this critical frequency corresponds to the locking range for the particular drive strength.

B. End-of-Lock-Range Dynamics

As well as stabilizing the mechanical oscillations, the injection signal can also be used to tune the oscillation frequency. The range of frequencies over which the mechanical mode can be locked has been studied extensively in the context of electrical tank circuits [16,18] and, more recently, optomechanical systems [30]. By definition, the oscillator is locked to the drive when the phase difference between it and the external drive, $\Delta \phi$, is stationary in time. When locked, $\Delta \phi$ is zero at the natural mechanical resonance
frequency, and grows with detuning to ±π/2 at the edges of the lock range, beyond which the oscillator fails to lock [16]. This change in phase with mechanical resonance frequency allows an injection-locked oscillator to be used for sensing applications where shifts in the mechanical resonance frequency are to be detected. This may offer benefits for sensor arrays compared with other approaches, such as using free-running regenerative oscillators or phase locking of driven oscillators, as stability is achieved without the need for individual feedback control circuitry to each sensor.

Here, we explore the dependence of the lock range on the two independent variables in our system, namely, (i) drive strength and (ii) optical power, and show good agreement with the recent theoretical study performed by Amitai et al. [30]. In addition to this, we find that previously studied end-of-lock-range dynamics occur only for sufficiently high optical power, with a new class of behavior evident at lower optical powers.

We begin by setting the system to regeneratively oscillate with $P_{in} = 23 \text{ mW}$ and applying an AC voltage of $V_{AC} = 0.5 \text{ V}$ at a frequency close to the mechanical resonance. With these settings, the amplitude of the mechanical oscillations due to the radiation-pressure force dominates the inertial drive by at least an order of magnitude. The oscillations successfully lock to the drive and $\omega_d$ is then increased (decreased) to find the upper (lower) end of the lock range. The end of the lock range is unambiguously signaled on the spectrum analyzer, marked by a characteristic spectrum that is representative of quasi-locking [18].

As discussed in Ref. [18], this regime of quasi-locking is characterized by the oscillator slipping in and out of lock. This manifests as $\Delta \phi$ alternating between periods of being held near ±π/2 and cycling through 2π radians before being locked near ±π/2 again (c.f., Supplement 1). The fraction of the time for which $\Delta \phi$ is held near ±π/2 decreases as the drive frequency moves further out of the locking range [18], until the oscillator is no longer locked at all. Supplement 1 covers the phase dynamics in greater detail. Here, it suffices to consider that this results in a phase difference which oscillates in time. This translates to a periodic frequency modulation, resulting in the sidebands that make up the triangular single-sided spectrum characteristic of quasi-locking [18], as shown in Fig. 5(a). The top (bottom) trace corresponds to the upper (lower) end of the lock range, whereas the two spectra in between are within the lock range.

For injection frequencies outside the lock range (not shown), we observe, as expected [18], *injection pulling* of the optomechanical oscillator. This term comes from the fact that, in this range of detuning, the resonance peak appears to be pulled toward the injection signal [18].

The locking phenomena discussed above are qualitatively the same as those observed for injection locking of electrical tank circuits [18], as well as a previous demonstration with an optomechanical system [26]. However, when these parameters are varied to obtain a larger locking range, we find that the end-of-range dynamics are strikingly different.

Decreasing $P_{in}$ to 10 mW with the same drive strength ($V_{AC} = 0.5 \text{ V}$) to obtain a larger locking range [30], we repeat the search for the ends of the lock range. With these settings, the amplitude of the mechanical oscillations due to the radiation-pressure force and due to the inertial drive are of the same order. We find that the end of the lock range is no longer marked by the distinct quasi-lock spectrum. Figure 5(c) shows the spectra near the lower end of the lock range, where the drive frequency is decreased from the top to bottom traces. The end of the lock range is now instead marked by an increase in power at the natural resonance frequency, rather than the quasi-lock spectrum. Outside the lock range here, injection pulling is not observed. We refer to this apparently distinct regime of partial locking as the “continuous-suppression” regime, as the peak at the mechanical resonance frequency appears to be suppressed as the injection signal moves into the locking range. The transition from the quasi-lock regime to the continuous-suppression regime occurs when the driven oscillation amplitude approaches or even exceeds the amplitude due to optical pumping. The crossover is not entirely well defined, however, as some settings result in one end of the lock range being marked by quasi-locking while the other being marked by continuous suppression of the natural peak.

In order to understand these two seemingly distinct locking regimes, we modeled the system using classical equations of motion
for the coupled optical mode (characterized by its light amplitude $\alpha$) and mechanical mode (characterized by its position $x$), which can be derived from the Hamiltonian given earlier:

$$\dot{\alpha} = -\frac{\kappa}{2}\alpha + i(\Delta + Gx)\alpha + A_L,$$

$$m_{eff}[\ddot{x} + \Gamma\dot{x} + \omega_0^2x] = hG|\alpha|^2 + F_d(t).$$

The mechanical mode, with a decay rate of $\Gamma/2\pi = 15$ kHz, is subject to both a radiation-pressure force and an electrical drive. We omit in our simulations thermo-optic effects, which affect the refractive index of the material [23], as well as the thermal Langevin force [6]. We numerically solve the coupled differential equations to simulate the dynamics of the system in the locking regimes explored experimentally in Figs. 5(a) and 5(c), and plot the results in Figs. 5(b) and 5(d), respectively. We find excellent qualitative agreement between the simulation and experiment, highlighting that these simple classical equations of motion capture all the experimentally observed locking dynamics, including the cross-over between the quasi-lock and continuous-suppression regimes.

**C. Quantifying the Locking Range**

In this section, we characterize the dependence of the lock range on the optical power and drive strength and compare our data with the predictions of Amitai et al. [30]. We first determine the largest lock range achievable with our device, by maximizing the output of the RF source to $V_{AC} = 5$ V and lowering the optical power to $P_{in} = 11$ mW. Figure 6 shows plots of traces of the power spectrum for varying drive frequencies, spanning the lock range for these settings, marked by green dots. The lock range achieved is 71 kHz, over $2 \times 10^6$ times the linewidth of the unlocked regenerative oscillations. While this range can be easily increased by increasing $V_{AC}$ or $V_{DC}$, this value already represents a tuning percentage of $\sim 1\%$. For applications which require arrays of optomechanical systems oscillating regeneratively at the same frequency, it is important to compare this tuning percentage to the expected variations in natural mechanical resonance frequencies. For the case of a circular optomechanical resonator, the resonance frequency of a radial breathing mode is inversely proportional to the radius, $\omega_{in} \propto 1/R$. Given that a 100 μm disk can be fabricated to well within 1 μm precision with standard lithographic techniques, the tuning range demonstrated here is already sufficient to overcome the <1% variations expected in $\omega_{in}$.

Referring back to the locked traces in Fig. 6, we find that setting the injection frequency of the strong drive ($V_{AC} = 5$ V) close to the mechanical resonance frequency causes the optical cavity to be shifted out of resonance such that regenerative oscillations cease. This is shown by the absence of traces marked by green dots between −10 kHz and 10 kHz. We suspect that this is due to thermal effects in the optomechanical system, as explained in Supplement 1. Here, we emphasize that this effect does not preclude locking over the entire range, as $V_{AC}$ can be reduced to lock near the natural mechanical resonance frequency. This is demonstrated by the power spectra marked by orange squares in Fig. 6(c), where $V_{AC}$ is reduced to 0.5 V to successfully lock near the center of the range.

Amitai et al. [30] show theoretically that the locking range $2\omega_r$ is directly proportional to the drive strength and inversely proportional to the amplitude of mechanical oscillations, $r_0$ (units of m). Adapting their expression to our parameters, we have

$$\omega_r = \frac{x_0^2 \delta C V_{AC} V_{DC}}{\hbar \delta x r_0}.$$ 

The amplitude of regenerative oscillations, $r_0$, is a function of the optomechanical parameters of the system as well as $P_{in}$, which can be controlled in the experiment. We first determine the lock range as a function of $P_{in}$, as explained previously, two regimes of end-of-range dynamics are observed in the experiment. In one, $\omega_r$ can be unambiguously quantified owing to the appearance of the quasi-lock spectrum. While some hysteresis was observed depending on the direction of the frequency sweep [29], its effect on quantifying $\omega_r$ was minor and thus neglected. In the continuous-suppression regime, however, the end of the lock range is marked by a relatively gradual re-emergence of the peak at the natural frequency $\omega_{in}$. For consistency, we choose to define $\omega_r$ in this regime to be the frequency offset at which the peak at $\omega_{in}$ reaches 30 dB below the peak at $\omega_d$. We note that this choice of threshold does not have a significant effect on the results as the peak at $\omega_{in}$ rises from being completely suppressed (below the noise floor) back to its original amplitude when $\omega_d$ is varied over a frequency offset range corresponding to ~5% of the lock range.

Figures 6(c) and (d) show plots of the measured locking range $2\omega_r$ as a function of $P_{in}$ for $V_{AC} = 2.5$ V (green squares) and 5 V (yellow circles). While the data include measurements in both the quasi-lock and continuous-suppression regimes, the data points nevertheless follow clear trends. In order to compare these results with locking ranges predicted by Eq. (6), we require the regenerative oscillation amplitude $r_0$, which is not precisely determined in the experiment. We thus use the equations of motion [Eq. (5)] to calculate $r_0$ in the absence of an inertial drive. Figure 7(b) shows plots of $r_0$ as a function of optical power $P_{in}$ for detunings of $\Delta/2\pi = 42$ MHz, 44 MHz, and 50 MHz, illustrating the sensitivity of $r_0$ to both these parameters. We find that, when inserted into Eq. (6), $r_0(P_{in}, \Delta)$ for $\Delta/2\pi = 44$ MHz (solid line)
produces the best fit to the measured locking ranges. The fit is plotted as a solid curve in Fig. 7(a), demonstrating good agreement between theory and experiment. Note that a free scaling parameter $\beta = 0.65$ is added to Eq. (6), to account for uncertainties in $\delta C/\delta x$ and $x_{ir}$. The fit could be further improved by taking into account the modification of the effective optical detuning with $P_{\text{in}}$. Increasing $P_{\text{in}}$ modifies the effective cavity resonance due to the steady-state radiation-pressure-induced cavity enlargement. This dependency is naturally corrected for in the experiment by always tuning the laser frequency to maximize the regenerative oscillation signal. However, for simplicity, the calculations for Fig. 7(b) were performed with a fixed laser frequency. Adding this correction, which we estimate to result in a shift of a few MHz in detuning over the range of laser powers used in our experiments, would reduce the steepness of the calculated curves at lower laser power, resulting in a closer match to the experimental data. As confirmation of this mechanism, we include the calculated locking ranges for $\Delta/2\pi = 42$ MHz and 50 MHz to Fig. 7(a), which enclose all the experimental data.

In contrast, the measurement of locking range as a function of $V_{\text{AC}}$ does not involve changes in the optical detuning and can thus be more straightforwardly compared to the theory. Figure 7(c) shows plots for this for $P_{\text{in}} = 12$ mW (red squares) and 16 mW (blue circles), confirming the linear dependence predicted by Eq. (6).

4. CONCLUSION

We have reported the first observation of locking of the radiation-pressure-driven regenerative oscillations of an optomechanical system to a direct inertial drive. We demonstrate a suppression of over 55 dBC/Hz of the phase noise of the regenerative oscillations at 2 Hz and a locking range of 71 kHz, more than 2 million times the 30 mHz oscillation linewidth. This tuning range is sufficient to overcome variations in natural mechanical resonance frequencies due to limits in fabrication precision. Applications requiring distributed optomechanical systems or arrays of oscillators within a single chip may benefit from injection locking. Although phase-locked loops can also be used to stabilize the mechanical resonance frequency, they require feedback circuitry, which may prove impractical for large arrays of devices. The feedforward nature of injection locking allows stabilization to be achieved without feedback circuitry. Moreover, the electrical drive presented here can be implemented for arrays more efficiently than optical injection, which requires either additional lasers or optical power modulators. This is especially true in the case where multiple oscillation frequencies are desired.

At a system level, the direct inertial drive we implement is distinct from that used in prior explorations of injection locking in optomechanical systems as well as on other physical platforms. To our knowledge, the only previously known difference between the two drive forms is the absence of harmonic locking effects for the direct drive [29]. We present previously unreported locking dynamics as described by the continuous-suppression regime, enabled by the ability to increase the inertial injection signal to approach and even exceed that of the radiation-pressure force. Further research is required to explain the underlying mechanisms responsible for the cross-over between the two regimes with distinct locking dynamics. Recently, Tóth et al. [43] also demonstrated locking with an injection signal which bypasses the nonlinearity in a radiation-pressure electromechanical system. This experiment explored the reverse regime to that studied here, with the mechanical oscillator driving regenerative oscillations of a microwave field, allowing injection locking of the microwave resonance.

Beyond applications in sensing, the electrical drive technique introduced in this work may enable the possibility to engineer optomechanical gain competition. For instance, we observed that for some devices, non-ideal circularity split the radially symmetric mechanical mode into two quasi-degenerate modes separated by $<1$ kHz, as visible in Fig. 5(c). Upon application of the drive to the resonance frequency of either mode, the system could be made to oscillate regeneratively on the targeted mode. Indeed, much like for the optical modes of a laser, different mechanical modes compete for gain in an optomechanical resonator [32]. We provide a detailed numerical analysis of this phenomenon in Supplement 1 and confirm that the electrical drive can reorient the oscillator along a new stable trajectory. The switch of optomechanical gain from one mode to the other persists even once the drive has been turned off, potentially serving as a form of “non-volatile” optomechanical memory [44] or enabling the
exploration of normally inaccessible stable dynamical attractors of the system [45,46].

**Funding.** Australian Research Council (ARC) (CE110001013, FT140100650, LP140100595); University of Queensland (UQ) (UQFEL1719237).

**Acknowledgment.** This research was primarily funded by the Australian Research Council and the Lockheed Martin Corporation through an Australian Research Council Linkage Grant. Support was also provided by a Lockheed Martin Corporation seed grant and the Australian Research Council Centre of Excellence for Engineered Quantum Systems. W. P. B. and R. K. acknowledge fellowships from the Australian Research Council and the University of Queensland, respectively. This work was performed in part at the Queensland node of the Australian National Fabrication Facility, a company established under the National Collaborative Research Infrastructure Strategy to provide nano- and microfabrication facilities for Australia's researchers.

See Supplement 1 for supporting content.

**REFERENCES**